AD-A125 898 REPRESENTATION OF MULTIVARIATE FUNCTIONS(U) TEXAS UNIV AT AUSTIN DEPT OF MATHEMATICS E W CHENEY 14 FEB 83 ARO-16952.4-MA DAAG29-80-K-0039 1/1

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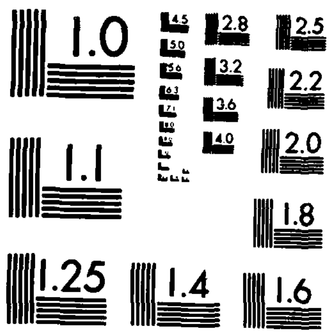

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This research project concerned the general problem of how to represent (exactly or approximately) a given mathematical function of several real variables. Scientific computations often involve multivariate functions (i.e. functions of several variables) For efficiency in computing, these functions must be represented in a form which can be rapidly computed (to whatever precision is demanded by the task at hand). This usually means that the given function will be replaced by an easily-computed approximate function. Once an appropriate form has been selected for the approximating function, the task of determining the best function of that form arises. Also there are questions of existence of best approximations.

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FINAL REPORT

Agency:	U.S. Army Research Office Box 12211 Research Triangle Park, NC 27709
Grant Number:	DAAG29-80-K-0039
Grant Period:	1 July 1980 to 31 October 1982
Principal Investigator:	E.W. Cheney Department of Mathematics The University of Texas Austin, Texas 78712
Title of Grant:	"Representation of Multivariate Functions"
Date of This Report:	14 February 1983

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Once an appropriate form has been selected for the approximating function, the task of determining the best function of that form arises. Also there are questions of existence of best approximations.

The research supported by this grant has led to eight technical papers which are in various stages of the publication process. The remainder of this report will give brief descriptions of these technical papers.

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Technical Papers

1. W.A. Light and E.W. Cheney, "Some proximinality theorems in tensor-product spaces". CNA Report 165, October 1980, University of Texas. Published in the Mathematical Proceedings of the Cambridge Philosophical Society 89(1981), 385-390. Math. Reviews 82d : 41045.

   In this paper, we are concerned with approximating a given function f as follows $f(x,y) \approx \sum_{i=1}^{n} u_i(x)h_i(y) + \sum_{i=1}^{m} v_i(y)g_i(x)$ where the base functions h_i and g_i are given, and the coefficient functions u_i and v_i are to be chosen. Five basic existence theorems are given which assert (in various situations) the existence of optimal choices for u_i and v_i. For examples, if X and Y are σ-finite measure spaces, if $g_1, \dots, g_m$ belong to $L_1(X)$, and if f belongs to $L_1(X \times Y)$ then optimal elements v_i exist in $L_1(Y)$ to minimize

   $$\int_Y \int_X \left| f(x,y) - \sum_{i=1}^{m} v_i(y) g_i(x) \right| dxdy \ .$$

2. E.W. Cheney and J.R. Respess, "Best approximation problems in tensor product spaces". CNA Report 167, January 1981, University of Texas. To appear in Pacific J. Math.

   This paper contains twenty new theorems on the approximation problem

   $$f(x,y) = \sum_{i=1}^{n} u_i(x)h_i(y) + \sum_{i=1}^{m} v_i(y)g_i(x) \ .$$

   As an example, if $\{g_1, \dots, g_m\}$ forms a Tchebycheff system on $[a,b]$ and if h_1 is a positive continuous function on $[\alpha,\beta]$, then each continuous function f on $[a,b] \times [\alpha,\beta]$ has a best approximation of the above form, with $n = 1$, and with continuous coefficient functions. Many of the results are stated for more general spaces, with a variety of norms.

3. E.W. Cheney and J.R. Respess, "Approximation problems in tensor product spaces". In Approximation Theory III, pages 729-734, E.W. Cheney, editor, Academic Press, 1980.

(This is a summary of paper #2.) MR82B : 41040.

4. E.W. Cheney and M. Von Golitschek, "Failure of the alternating direction algorithm for best approximation of multivariate functions". CNA Report 172, May 1981, University of Texas.

This paper shows that a promising algorithm for approximation problems of the form

$$f(x,y) = \sum_{i=1}^{n} [u_i(x)h_i(y) + v_i(y)g_i(x)]$$

works only in the case $n = 1$, if the domain of f is a rectangle in the plane and if the base functions g_i and h_i form Chebyshev systems.

5. E.W. Cheney and J.R. Respess, "On Lipschitzian proximity maps", CNA Report 179, July 1981, University of Texas. In Nonlinear Analysis and Applications, edited by S.P. Singh and J.H. Burry, pages 75-85. Lecture Notes in Pure and Applied Mathematics, vol. 80, Marcel Dekker, Inc., New York, 1982.

This paper concerns "proximity" maps, which are nonlinear operators designed to produce best approximations in prescribed sets of approximating functions. For example, if f is a continuous function on a closed finite interval, and if a positive integer n is given, then there exists a unique polynomial p of degree at most n which deviates least from f on the interval. The mapping A which produces $p = Af$ is a proximity map. In numerical problems, it is advantageous to know that A is continuous. In more complicated situations, additional properties of A, such as $\|Af-Ag\| \leq k\|f-g\|$, are needed. Such properties are studied in this report.

6. E.W. Cheney and W.A. Light, "The characterization of best approximations in tensor product spaces". CNA Report 175, September 1981, University of Texas. To appear in the International Journal of Analysis.

In this paper, a theorem is proved which gives a necessary and sufficient condition for a function of the form

$$\phi(x,y) = \sum_{i=1}^{n} [u_i(x)h_i(y) + v_i(y)g_i(x)]$$

to be a best approximation to a function $f(x,y)$. Here f, g_i, and h_i are prescribed continuous functions, and the coefficient functions u_i and v_i are chosen to make $|f(x,y)-\phi(x,y)|$ uniformly as small as possible.

7. E.W. Cheney and M. Von Golitschek, "The best approximation of bivariate functions by separable functions", CNA Report 179, October 1982. To appear in Proceedings of a special session of the American Mathematical Society held at Toronto, August 1982, S.P. Singh, editor (published by the American Mathematical Society).

A "separable" function of two variables is one which can be written in the form

$$\phi(x,y) = \sum_{i=1}^{n} g_i(x)h_i(y) .$$

They arise in the study of integral equations and partial differential equations. This report is concerned with the problem of approximating an arbitrary function of two variables by a separable function, within the setting of spaces of continuous functions. It contains nine new theorems. One of them, for example, states that if f, g_i, and h_i are fixed continuous functions, then in the uniform approximation of $f(x,y)$ by $\sum_{i=1}^{n} [u_i(y)g_i(x) + v_i(x)h_i(y)]$ one cannot achieve higher precision with bounded coefficient functions u_i and v_i than with continuous coefficient functions.

8. E.W. Cheney, "Survey of approximation of multivariate functions by combinations of univariate ones". (Manuscript to be published in Proceedings of the Approximation Conference, Texas A&M University, January 1983.)

This is a survey of what is known and what is unknown in the problem of approximating multivariate functions by simpler functions which are built up from univariate functions by addition, multiplication, composition, and the taking of limits.

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